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FOREWORD TO THE TRANSLATION TO ZILLMER'S PAPER

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The full preliminary term reserve was invented by the German actuary Dr. A. Zillmer (a President of the German Life Insurance Institute) in this 1863 paper, and by Thomas B. Sprague (a President of both the Scottish Faculty of Actuaries and the Institute of Actuaries) in an 1870 paper (JIA), and later in 1895 in a paper in the Proceedings of the First International Congress of Actuaries, p 7.1. Although Sprague made significant contributions to the theory of reserves, Sprague gave Zillmer the credit for the invention of the preliminary term reserve.

To this day, in Europe preliminary term reserves are often referred to as "Zillmerized reserves."

Unfortunately, the original Zillmer paper was written in German (actually in German script), and I could find no complete translation of the paper, although a few paragraphs were translated in a 1902 paper "Some Notes on the Net Premium Method of Valuation...", JIA 37, p.57, by Samuel George Warner.

Around 1985 with the help of North American Reassurance and its parent Swiss Reinsurance in Zurich, I was able to obtain a copy of the original paper. The Guardian Life paid for a translation of the paper, and with significant help from John Cole, FSA and Court Smith, FSA I was able to obtain a fairly good translation.

The paper is divided into two parts; in the first part Zillmer shows how one can include a provision for first year commissions in the reserve calculation. In the mid nineteenth century commissions paid by German companies were apparently 1-1.5% of the sum insured, or about ½ the typical gross premium; but the ratio of commission to premium varied by age. Therefore, negative reserves could develop, particularly at the younger ages. Zillmer suggested companies replace the negative by zero, and carry forward the unamortized allowance to future policy years.

In the second part of the paper Zillmer discusses the evils of the gross premium reserve, which tends to upfront profits. In Zillmer's day the loading on whole life policies was 30%-40% or more of the net premium, and the gross premium reserve had the effect of massively up fronting profits.

Zillmer also describes an interesting method of reserving which I have not seen discussed elsewhere. (Zillmer disfavors the method, but points out it is much more conservative than gross premium valuation.) Thus, as long as the gross premium is higher than the attained whole life premium, the company takes the net premium to be a one-year term premium, with the excess reckoned as profit. After 5-7 years the net premium would equal the gross, and the company would have to post a reserve.

The monetary units in the paper are in "thaler" and a single thaler in 1861 was worth \$.75 in U.S. currency.

The translation uses the word “bank” -- but “life insurance company” could also be used. We have added emphasis to various points -- using boldface. The emphasis was not found in the original paper. The paper defines the “full preliminary term method” on pages 12-15, wherein the minimum reserve at year 1 or at year 2 is set at zero.

Another translation of the Zillmer paper was published in the 1990 and a copy was found on the web.

Besides its obvious historical interest is the paper significant today? . Perhaps. In 1990 the United States imposed a “DAC tax” on life insurance and group life and annuity policies under section 848 of the Internal Revenue Code. (“DAC” is for deferred acquisition cost) . Many experts felt that the DAC overcounted the acquisition costs, since most or all of the acquisition costs are deferred using Zillmerized reserves which are used in computing federal tax reserves. (Zillmerized reserve include the modified or Preliminary Term and Commission Reserve Valuation Method (“CRVM”) reserves). The paper clearly indicates that the purpose of Zillmerized reserves was to defer some or all of the acquisition costs. The tax authorities, however, have been skeptical that the CRVM reserve defers some acquisition costs.

We have added certain comments in brackets [].

CONTRIBUTIONS TO THE THEORY OF THE PREMIUM RESERVE FOR LIFE INSURANCE COMPANIES

BY

Dr. A. Zillmer
Actuary of the Germania Life Insurance Company in
Stettin, 1863
Press of Theodore von der Nahmer

[ms.p1]

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This paper is divided in two parts. In part 1 the paper discusses how closing costs or sales commission can be included in the reserve calculation. In part 2 Zillmer outlines the problems with the gross premium reserve method.

To this day Europeans refer to preliminary term reserves as “zillmerized reserves.”

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[ms. P 3]

TABLE OF CONTENTS

Introduction	Page 5
I. About the Reserve and the Sales Commission	Page 7
II. About the Calculation of the Reserve based on Gross Premiums	Page 23

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[ms. P5 & 6]

INTRODUCTION

“Life insurance companies should be held to more precise and stringent accounting standards than any other corporations. Joining a life insurance company is an act of self-abnegation and a result of the noblest love of wife and children. Often the life insurance policy is the only inheritance, the only resource for widows and orphans. Since life insurance companies collect a considerable portion of the savings of thousands of insureds on the basis of contracts some of which do not expire until 50 or 100 years later, the security and the liquidity of the companies must be mathematically demonstrable and be incontestable.”

We want to apply these irrefutably true words, which were spoken by Chief Commissioner of Insurance of the State of New York, when the right to do business in New York was withdrawn from the New Haven Life Insurance Company (Connecticut), in the following observations about two difference methods of calculating the premium reserve for life insurance companies.

Although Wilhelm Lazarus already made us aware in 1857 in a thorough article (in the Rundschau der Vergicherungern (page 317) of the false principle of basing the present value of assumed liabilities on a low interest rate, rather than on the present value of the [net] premiums still to be collected, and show the consequence of such a false valuation on the balance sheet of a company using such a method, almost nothing has happened since then to stop the spread of this misconception. [Unclear translation the prospective gross premium reserve produced a “low reserve” because the gross premium was greater than the “net.”] Occasionally, one finds in professional journals the comment that the reserve calculated on the basis of gross premium is too small. But that is all. Just recently, as a result of the published balance sheets of some English companies, some voices have been raised once again against this ill-conceived reckoning. In addition, other aspects of the insurance business have been drawn into the discussion and a few misconceptions have arisen. For example, it has been claimed that it is impossible to maintain an adequate amount of reserve given the new custom of paying an agent a high sales commission for a policy, about 1% of the total sum insured, and thereby companies which pay such **sales commissions** are forced to set their reserve too low – even if not according to the gross premiums. Such misconceptions, even if they are not intentionally meant to deceive, serve to shatter the public’s confidence in life insurance companies and harm precisely those institutions, which most openly publish details of their business and make no secret of their amount of reserve. We shall try to disseminate correct views about the premium reserve of life insurance companies, as far as possible, and present in the first section of this paper the proof that an adequate reserve can be maintained even if closing commissions are present. In the second part, we will briefly discuss the method of valuing with gross premiums and believe we will thereby negate some of the false aspects which are a natural consequence of this misconception.

[ms. P7 - 13]

I. RESERVES AND CLOSING COMMISSIONS

The premium reserve plays a large role for life insurance companies. The yearly premium for a policy – we are thinking of an ordinary life insurance policy with annual premiums – remains the same for the duration of the policy, while the risk increases every year that the total value will come due as a result of the insured’s death and, consequently, if the insurance were bought a year at a time, the premium would increase every year. Therefore, the constant level premium applying for the duration of the policy is in the early years larger and, the later years smaller, than the premium for a year of insurance. If a life insurance bank takes on a certain number of people, then in the early years the bank will set aside from their premiums a considerable sum of money not consumed by the insured’s deaths, and this amount which is set aside must then cover the claims which the bank must pay out later as the number of deaths increases and the amount that must be paid out is higher than the amount of the premiums being collected. Table II shows that a bank which has insured a number of 30-year-olds for 500,000 Thaler takes in more money in premiums during the first 25 years than it pays in death claims. Starting with the 26th year, the amount of the death claims exceeds the amount being collected in premiums. For example, in the 40th year, the difference equals 8679 Thalers.

Therefore, as a necessary consequence of the establishment of the premium, a life insurance bank must set up a reserve. The amount of the reserve cannot be determined arbitrarily by the bank, but must be calculated with mathematical precision like the [net] premium based on actuarial tables and a fixed interest rate. For example, the insuring bank may not decrease the reserve fund because it counts on a chance to keep the number of deaths below the number expected based on

the actuarial tables or expects favorable speculation or related business to result in higher interest income so that it can offset the decrease in the reserve fund. Rather, the reserve fund should be based on the insureds' premiums accumulated at interest after subtracting the expected disbursements for death claims and in fact the expected disbursements, not the actual disbursements, because otherwise, as a result of a continually excessive mortality among insureds, the reserve funds, if not completely exhausted, would limit the bank's prospects considerably. Moreover, the reserve fund should not be established from the [gross] premiums paid by the insureds, but rather on the net premiums. The premiums as determined according to actuarial formulas (i.e. the net premiums), are increased by a surcharge to compensate the bank for expenses and for the risk that the number of deaths will exceed the expected. This surcharge, which is set more or less according to the needs of each individual bank, accrues to the bank, with the reserve fund being based on the net premium remaining after this surcharge is deducted. As soon as the net premium for an insurance policy is established according to the foregoing procedure, the amount of the reserve is given mathematically for any point in time within the term of the insurance.

Usually, the net premium will be fixed so that like the gross premium payable by the insured it will remain constant for the duration of the policy, but obviously there is no reason not to decide to set the net premium according to a different schedule as long as the formula is rational and the result is such as would completely cover the bank's liabilities. There are also insurance policies with increasing or with decreasing premiums rates where not only the gross premium but also the net premium varies as between different periods of the term of insurance.

Let me preface with a few observations my presentation of the method for establishing net premiums which is generally preferred and which I believe must be utilized given the present state of the insurance business.

The German public's participation in life insurance has increased dramatically in the last few years. Not only are some of the older German companies doing considerable business as compared with previous decades, but also now a few of the newer German companies are achieving outstanding growth. Germania has found so much favor that in the previous year (1862) sales amounted to much more than 6 million of insurance, and in this April alone reached more than a million. The glowing success of the life insurance business is not only the consequence of increasing recognition by members of the public of the true value life insurance represents for the individual family and for the State. This success is also the result of conscientious work by the management of life insurance companies and by their agents. As a result of giving the agent a greater closing commission for selling insurance – whereas in the past, he only received a constant service fee for collecting the premium and only accumulated significant income after years of labor – it became possible to attract aggressive businessmen as agents, who could give up their other businesses and devote their entire energy to the sale of insurance. Despite the success realized by the introducing of this commission practice, a few voices have been raised against it. ¹

Instinctively, the question is how a company can pay an agent huge commissions if there is a huge increase in sales. We concede, that if companies are not only required to maintain an adequate reserve – this must of course be stipulated – but are also required to base the amount of the reserve on an antiquated methodology, then a grave situation can result. The **expenses of acquiring an insurance policy**, for covering mortality in the first year, and for setting up the reserve at the end of the first year can be, and under certain circumstances must be, greater than the premium collected in the first year.

The resulting deficit would naturally increase according to the increase in new business. And if the increase in business is great enough to strain the resources of a company, even a layman must recognize that a contradiction arises. On the one hand, the acquisition of new business is desirable; a company can suffer financial distress from new business even if the increases are not too high—as we will show.

It would be strange if we not only could find a way out of this dilemma, or if by the nature of the problem, this were not possible. And, in fact, there is a way out. Keeping to the usual methods of booking insurance sales sets a company back with growing costs, so that we are led to the notion of an account for commissions paid in advance, with **this account being an asset** of the bank. The amount booked in the account each year must either be amortized within a certain number of years or, what would be preferable, every year a fixed and not too low proportion of premiums collected must be written off against the total advanced commissions. This practice has already been openly discussed in an article from Breslau in the Deutsche Versicherungszeitung of 26 April, 1863. For several reasons, we cannot accept its views. **First** of all, there are no formulas or limits for the amount of costs to be booked in this account. Essentially, this account would have the effect of diminishing either the capital or the reserves. The possibility exists that a company's financial security could be endangered by excessive expenditures for new policies. **Second**, if the artificially created asset for advanced commissions is actually amortized in full, a relatively larger portion of premium income is required each year and the net income available to cover other expenses will shrink unnecessarily for a great number of years. **Third**, advanced commission can and will remain in the account even though disbursed for insurance policies that have since lapsed. Future insureds then have to cover costs incurred by previous insureds. ² [U.S. GAAP accounting does treat acquisition costs as an asset.]

These problems can be avoided in the following easy way. The net premium for a sum insured is fixed so as to be a precise amount smaller in the first year than in the ensuing years.³ We will pass over the derivation of the applicable formula here, which is, by the way, quite simple. The **adjusted net premium** is derived from the usual net premium (which remains constant for the duration of the policy) by dividing the amount by which the first year net premium should be less than the renewal by the amount of the annuity due for the insured's issue age and then adding the result to the usual net premium. For example, according to the mortality table of 17 English Insurance Companies and the usual net premium for insurance to age 90 (which we will use throughout this article), the premium at issue age 40 is 2.484%. If the first-year net premium should be smaller than the renewal by 1% of the sum insured, then, the renewal net premium is 2.484%, plus 1% divided by the 50-year annuity due issued at age 40, i.e. $2.484\% + 0.059\% = 2.543\%$, and the first-year net premium is $2.543\% - 1\% = 1.543\%$. But if the first-year net premium should be smaller than the renewal by $1\frac{1}{4}\%$ of the insured sum, then the first-year net premium is $2.484 + (5/4 \times 0.059) - 1.25 = 1.308$ and the renewal net premium is $2.484 + (5/4 \times 0.059) = 2.558$.

Of course, the insured pays the same premium in the first year as he will pay in the ensuing years and the bank defrays the cost of acquiring the insurance policy with the extra portion of the premium at its disposal in the first year. The reserve is calculated the same way as with the usual net premium, i.e. the reserve for an insurance policy that has been in force a number of years is calculated by subtracting from the single premium at the higher age the product of the net premium and the annuity due at the higher age. The reserve will be smaller than it was using earlier methods, thus fulfilling the requirements exactly:

1. in that it is constituted only from the net premium after deducting the expected cost of mortality and adding the assumed interest, or

[ms page 12-13]

2. (in other words) in that together with gross premiums payable in the future by the insureds (which means net premiums too) the financial obligations of the company are covered completely.

To compensate for the smaller reserve than in the old method, the net premiums are somewhat, but not significantly, higher.

Before we consider the reserve further, we want to say something about the first-year premium. A **minimum premium** applies for every age at issue and this is **determined by the premium for a year of insurance**. If the premium for the first year is much smaller than the premium for one-year of insurance, then the mortality in the first year would not be covered fully by the net premiums, the insurance company would have to defray the excess mortality by other means. Just as the minimum for the first-year net premium is given by the premium for a year of insurance, at the same time the maximum is given at each issue age for the difference between the first year net premium and the renewal net premium.

If we call the difference X , then obviously the usual net premium, increased by X divided by annuity due, must be equal to the premium for a year of insurance, increased by X . If we solve the equation, then X , (i.e. the maximum allowance for sales costs), equals the difference between the usual net premium and the premium for a year of insurance multiplied by the value of the annuity due at the issue age and divided by the same annuity minus one, or by the immediate annuity at the same age).

The expression of the maximum closing commission can be transformed many ways. For example, this maximum is equal to the result of deducting one from the quotient of the annuity at the issue age and the annuity at the next higher age (i.e. for the insurance amount one), or equal to the difference between the usual net premium at the next higher age and the usual net premium at the issue age multiplied by the annuity at the issue age, or also, and this is the simplest expression, the maximum closing commission is equal to the usual net premium at the next higher age, reduced by the premium at the next higher age, reduced by the premium at the issue age for term insurance of one year. This simple form for the maximum sales commission can also be expressed as follows. The maximum the company can afford for acquiring a policy is clearly reached if the reserve is zero at the end of the first year. The latter is the case if one takes as net premium the usual net premium at the next higher age. Since the premium for the term insurance is used to cover mortality, then what remains to cover sales costs is at the most the difference between the usual net premium at the next higher age and the premium for term insurance at the issue age.

The following table shows the maximum rate for sales costs for various ages, if calculations are made according to the table of the 17 English life insurance companies at a 3 1/12% discount.

Issue Age	Maximum Rate of Closing Costs as % of the insured sum
20	0.71
25	0.80
30	1.04
35	1.27

36	1.32
37	1.38
38	1.44
39	1.50
40	1.58
45	1.95
50	2.33
55	2.77
60	3.28

[ms. P14]

It can be concluded from the table that when the average issue age is 37 or more something more than 1 3/8% of the insured sum can be used for sales commissions. However, if one keeps in mind that the maximum rate increases with age and that the younger issue age generally consists of a smaller insured sum as well, then one may well assert that an insurance company using the table of 17 English insurance companies at a discount rate of 3 1/2%, can put up 1 1/2% of the insured sum as the maximum which on average it can spend for the acquisition of a policy and to cover corresponding net premium.

But it does not follow that 1 1/2% of the insured sum must now be spent as closing costs. On the contrary, I consider it prudent to stay with 1% or 1.25% of the insured sum. It is possible that younger people may buy more insurance policies or that younger people may buy larger insured sums than has been experienced to date, which would lower the average issue age, or if the larger policies on average belong to younger people, the result of the calculation will look as if the average issue age has been reduced and then the calculated rate for the sales costs could turn out to be too high.

Since even where the sales costs are only 1% to 1.25% of the sum insured, one cannot absolutely protect oneself against the obvious risk of a reduction in the average issue age, I consider it necessary to take certain precautionary measures, as follows.

If one reckons sales costs to be greater than the maximum rate for the youngest issue age, then the net premium available for the risk at the younger issue ages is smaller than the premium for term insurance for one year at the corresponding issue age. Or in other words, the reserve at the end of the first year of insurance would be negative, while the net premium available in the ensuing years would be greater than the usual net premium for the next higher age. If the company then included the negative reserve at the younger ages in the total sum of reserves, then the total reserve will decrease precisely as much as the company was under the amount needed to cover expected mortality during the course of the year (increased with interest at year-end).

The higher the closing costs are reckoned to be, the larger the negative reserve will be and thus the more the total reserves will be reduced. It must be added, that the negative reserves released by lapsation of insurance policies after a year in force must be charged to the remaining business. In regard to these circumstances, I consider it necessary that a company not include negative reserves in the total sum of the reserves, but instead reckon them to be zero at the end of the first year. Here the company in any case incurs an expense exceeding the expected amount. This is easy to sustain as long as the expected sales costs are not set too high, as we will show. If the relationship changes so that the closing costs calculated so far seem too high, then this becomes evident in that the part of the income usually used to build profits is decreased, while the

premium reserve remains untouched, proving the company at the same time with a control over the amount of closing costs.

In the second year, the policies which would have a negative reserve at the end of the first year are included with the reserve, which takes into account the expected closing costs. If the reserve should still be negative at the end of the second year, then the value of the reserve is again posted as zero and the expected value is then included at the end of the third year. The latter occurs only with the lowest issue ages, as the following table illustrates:

[ms page 16]

RESERVE For INSURANCE of 100 THALER if it is Inforce

Issue Age	1 Year		2 Years		3 Years	
	1 st yr Premium		1 st yr Premium		1 st yr Premium	
	Full	Less 1.25%	Full	Less 1.25%	Full	Less 1.25%
15	0.583	-0.650	1.188	-0.047	1.800	+0.582
16	0.608	-0.674	1.233	-0.001	1.879	+0.652
17	0.620	-0.613	1.279	+0.640	1.947	+0.721

In general, the expense is not significant when the bank excludes negative reserves by setting them at zero, and thus maintaining for part of the business, reserves greater than necessary. If the closing costs are 1 ¼%, then the reserves at the end of the first year for issue age 35 are already positive. At 1%, the reserves at issue age 30 are positive. The younger ages are in general (the younger the age) less represented among insureds, so that where expenses are higher in individual cases, there are only a few individual cases that actually generate such higher costs. In addition, a business growing at a reasonable rate will only incur these expenses in the first year and then provide the positive reserve for the end of the second year. Since the bank already covers the negative reserve, these part becomes profit, or rather offsets the negative reserve for new insurance. With a growing business, the expenses would be repeated every year and also to the extent by which insurance at the younger ages grew over the previous year. To have a yardstick for the amount of these costs, we make the specific assumptions (which is certainly not too low) that an insurance bank insures in one year 10,000 Thaler for 20 years olds, 20,000 Thaler for 21 years old, 30,000 Thalers for 22 years old, etc., 100,000 Thaler for 29 years old, 120,000 for 30 years old, etc., and 200,000 for 34 year olds. Then if sales costs of 1% of the sum insured are calculated, the following results:

[ms. P17]

NEGATIVE RESERVE IN % OF			
Issue Age	Sum Insured	Sum Insured	Total
20	10,000 Th.	0.289	28.9 Th.
21	20,000 “	0.261	52.8 “
22	30,000 “	0.234	70.2 “
23	40,000 “	0.206	82.0 “
24	50,000 “	0.176	88.0 “
25	60,000 “	0.146	87.6 “

26	70,000 “	0.111	77.7 “
27	80,000 “	0.074	59.2 “
28	90,000 “	00.43	38.7 “
29	100,000 “	0.001	1.0 “
Total	500,000 Th.		586.1 Th.

If 1 ¼% of the insured sum is reckoned to be sales costs, then:

Issue Age	Sum Insured	Sum Insured	Total
20	10,000 Th.	0.873	57.3 Th.
21	20,000 “	0.512	102.4 “
22	30,000 “	0.482	144.6 “
23	40,000 “	0.453	181.2 “
24	50,000 “	0.424	212.0 “
25	60,000 “	0.391	234.6 “
26	70,000 “	0.359	251.3 “
27	80,000 “	0.322	257.6 “
28	90,000 “	0.289	260.1 “
29	100,000 “	0.249	249.0 “
30	120,000 “	0.211	253.2 “
31	140,000 “	0.178	249.2 “
32	160,000 “	0.128	204.8 “
33	180,000 “	0.082	147.6 “
34	200,000 “	0.033	66.0 “
Total	1,350,000 Th.		2,870.9 Thl.

[ms.p18]

If with a company there was 550,000 Thaler more insured each year over the previous year for issue ages under 30, then with 1% closing costs it would have excess expenses of only 586 Thaler, i.e. which does not reach ¼% of the premium income as long as premium income is more than 250,000. Or if sales costs are reckoned to be 1 ¼% then if there was 1,350,000 Thaler more insured each year for issue ages under 35, the yearly excess expense would be 2870 Thaler which is less than 1% of premium income if the latter is over 300,000 Thaler.

One might say that the company could reckon the maximum closing cost for every age. The total reserve for new insurance would then equal zero, and the net premium for each individual issue age could then equal the usual net premium at the next higher age. If a company calculates its premium from the beginning so as to add a specific percentage surcharge to the usual net premium at the next higher age, then the premiums for the higher issuing age become quite expensive. If it has calculated its premiums according to the usual method, then a relatively smaller proportion would be left over for administrative costs at the higher issue ages. (Moreover, the amount of agents' commission would have to depend on the issue age, which could lead to unpleasant consequences, as was already mentioned.) The following table shows the various percentages which a bank would have left over from the gross premium if its gross premium were calculated according to the usual method based on the table of the 17 English insurance companies at a discount of 3 ½% and a surcharge of 12 ½%. According to the usual method, 1/9 or 11 1/9% of the premium would go the company.

Administrative Costs in % of Gross Premium, if the First Year Premium is reduced by			
Issue Age	Maximum Rate	% of Sum Insured	1 ¼% of Sum Insured
20	8.94	8.04	7.28
30	8.46	8.55	7.01
40	7.81	9.01	8.49
50	7.14	9.41	8.00
60	6.53	9.71	9.36

[ms. P19]

In addition, the table shows that the agent's commission, which is determined by the earlier method according to a specified percentage of the premium, can be easily structured under the new method, so as to be more advantageous for the company. The agent will be glad to accept a smaller commission in the ensuing years if he receives a significant compensation in the first year after the sale. For example, if a company used to pay 6% of the premium to the agent as commission, then it only had 5 1/9% for other administrative costs. If it pays in addition a sales commission of only 2-3% of premium, then for most issue ages more than 5 1/9% is left over.

As regards the reserve for an insurance policy that has been in force a number of years, it is somewhat smaller than under the earlier method. The difference between the two reserves diminishes each year; as can be easily proven by mathematics, if the amount of the sales commission is reckoned to be a % of the sum insured, the difference is always a % of the sum insured less a % of the reserve under the old method. Since the reserve increases each year, the difference decreases each year. If the old reserve for an insurance of 100 Thaler is 25 Thaler, for example, then the new reserve with 1% closing costs is 21.25 Thaler:

25	21.25 Th,
59	49.50
75	24.75
100	100.00

The total reserve of an insurance bank for all insurance policies, as calculated according to the new principle, has, since each individual reserve is smaller, a smaller value than it would if calculated according to the old method. The difference will be larger if a larger number of insurance policies are relatively new. The difference here is also a % of the sum insured less a % of the reserve calculated by the old method, if sales costs are reckoned to be a % of the sum insured. The Gothaer Bank, for example, had insurance in force of about 38,800,000 Thaler and a premium reserve of about 8,700,000 Thaler at the end of 1861. If the Gothaer Bank had reckoned the closing commission to be 1%, then its reserve would be smaller by about (388,000 – 87,000) Thaler, i.e. 301,000 Thaler.

Table I attached shows the reserve for insurance of 100 Thaler at issue ages, 30, 40 and 50.

[ms. P20]

throughout the life of the insurance policy. One can see that the reserve based on a closing commission increases more rapidly than the old reserve, as must be so, since starting with the second year of insurance a higher net premium enters into the calculation. Tables II and III and IV show a group of people of the same age who insure themselves at the same time, and Table V gives a picture of the results an insurance company realizes if every year they insure anew such

business as Tables II, III, IV portray. It is seen here that after 60 years, for insurance in force of about 41,900,000 Thaler the old reserve would grow to 12,644,900 Thaler. The reserve based on 1% closing costs would be 12,352,200, and the reserve based on 1 ¼% closing costs would be 12,279,100. The difference between the old reserve and the new reserve according to the above formula is, with 1% closing costs, $419102 - 126449 = 292653$ Thaler and with 1 ¼% sales costs $(941902 - 12449) \times 5/4 = 365817$ Thalers.

Under the assumptions of Table V, after 60 years a kind of stationary in force is reached, i.e. as a result of deaths, the same sum insured becomes payable as that which is newly insured. As long as these assumptions hold, the sum insured, the premium income, and the reserve remain constant in the future. Under such circumstances, obviously the net premiums and the interest on both the net premiums and the reserve capital must together be as great as the yearly sum insured payable.

Of the aforementioned insurance in force after 60 years 16,947,500 Thaler is due to members entering at 30, 20,055,100 to members entering 40, and 4,907,600 to members entering at 50.

If to the insurance in force one adds new insured sums and calculates premium income according to the following formula:

Issue Age	Net Premium, if the First Year Premium is reduced by 0	1 Thaler	1 ¼ Thaler
30	1.7972	1.8490	1.3619
40	2.4842	2.5423	1.5576
50	3.7082	3.7791	3.7968

[ms. P21]

then one obtains the following where the income of the bank covers the outgo:

1. if the reserve is calculated excluding sales costs:

Net Premium	1,021,661 Thaler
3 ½% interest thereon	35,758
3 ½% interest on the reserve	<u>442,547</u>
Total	1,499,966 Thaler

2. when the reserve is calculated based on 1% sales costs:

Net Premium (less 15,000)	1,031,547 Thaler
3 ½% interest thereon	36,104
3 ½% interest on the reserve	<u>432,327</u>
Total	1,499,978 Thaler

3. based on 1 ¼% sales costs:

Net Premium (less 18,750 Thaler)	1,034,019 Thaler
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3 ½% interest thereon	36,190
3 ½% interest on the reserve	429,768
Total	1,499,977 Thaler

The smallest difference of 34,22 and 33 Thaler, respectively, in respect to the considerable sum of 1,500,000 Thaler is the result of rounding in Table V. (Thus confirming the long duration of the different reserves)

We can safely conclude from the above that a reserve based on a not too high closing cost is completely adequate for a bank's obligations if the insurance policies stay in force long enough. If an insured lets his insurance policy lapse, then the bank, if it has adopted the precautionary measure of booking the negative reserves at the younger issue ages not as calculated but rather as zero, and if the agent does not receive the corresponding amount of the closing commission or must return it if the policy lapses during the first year, no disadvantage should result. On the contrary, the reserves accumulated from such policies are released to the bank as profits. If insurance when terminated has stayed in force so long that the insured can claim surrender value, then it would naturally be based on the lower reserve.

As far as we know, no bank need pay the full amount of the reserve in such cases, rather only a specified proportion. For example, if the company makes it the rule to pay ¾ of the reserve, then it can do so, as soon as the reserve based on 1% closing costs grows to over 3 Thaler or based on 1 ¼% to over 3 ¾ Thaler for a sum insured of 100 Thaler, or the reserve excluding closing costs grows to over 4 or 5 Thaler, respectively, in that instead of ¾ of the reserve it grants a specified higher proportion of the same, paying so much as if the reserve were calculated excluding closing costs. And it can do so more easily, since it has already made good for all closing costs.

[ms. P23]

II. Gross Premium Reserves

In his previously mentioned article "The Security of Life Insurance Companies" (Insurance Rundschau[Review], 1857 year, page 317), Wilhelm Lazarus called attention to the fact that in the closing accounts of the Gresham Life Assurance Society for 12 November 1855 the liabilities of the company (even though it had been active but seven years) toward its insureds were estimated to be less than the value of future premium income, and yet a not insignificant amount of the premiums needed to cover the liabilities had already been received. A similar situation appeared in the balance sheet of the Great Britain, published a while ago. After almost 20 years of existence the Great Britain has instead of a premium reserve, an asset for the value of the future premium income of 529,469 and a liability for the value of sums insured of 494,707.

Therefore, not only is there no established reserve, but the company also included in assets, a profit 34,762 or 23,700 Thaler, which sum is the present value of the excess of what the company will earn from its present insureds, over what it expects to pay them after all claims are settled.

Now it is known that in the formula for the net premium the obligations of the insured are equated with the obligations of the bank. If one operates further with the net premium in the reserve formula, then the reserve can, under certain circumstances, as the first section shows, the reserve for a portion of the insurance issued during the previous year can, under certain circumstances, be

negative, but this negative result has little significance in regard to the reserves of the entire number of policies issued in the same year and [almost] nonexistent in regard to the total reserve of all policies issued in all years. It is therefore impossible that by the net premium formula the reserve could take on the value it does in Great Britain. In fact, not a small number of English companies calculate their reserve such that the future premiums payable by the insureds are not based on net premiums but rather on much higher amounts, and as a rule on gross premiums, while the liability of the company is based on net premiums. As a consequence of this calculation, the reserve no longer meets the requirement that it be composed solely of the risk premium remaining after deduction of payments covering expected mortality with the addition of assumed interest. This method of calculating the reserve is wrong.

Seemingly, the same formulas are used to calculate it as the net premium method. One subtracts from the value of the insured sum (the single premium for the insurance) the value of the premiums to be paid in the future, which the exception that one takes the gross premium, instead of the net premium. Herein lies the error. The reserve formula is only correct (if it is also derivable directly) provided it is a transformation of the other formulas where the reserve is composed of that part of the premium not used to pay death claims accumulated at interest.

Should a company wish to enjoy as soon as possible the ever increasing profitability of insurance sold, only the following way is available, if one wants and is capable of this acceptable procedure so long as the gross premium an insured pays is higher than the net premium at the ever increasing attained age of the insured, the company takes the net premium to be the premium for term insurance of one year, with the excess being reckoned as profit. But once the insured has reached the age at which the issue age net premium for whole insurance equals the gross premium being paid by the insured, then the full premium will be charged.

During the (5-7) years, using this method, no reserve would be established and only later the company would have a certain part of premium put into the reserve. Even with this almost unthinkable method, one cannot speak of a negative reserve; a new proof that the method used by English insurance companies in calculating the reserve with the gross premium is wrong.

[ms page 24-25]

The way just described of enjoying as soon as possible the profitability of an insurance policy is still outdone by the gross premium method. There the moment an insurance policy is sold a profit appears that amounts to as much percent as the combined value of the future premium payments as the percentage loading in the premium, or, since the value of the future net premium payments is equal to the single premium for the insurance, a profit that amounts to as much percent of the single premium as the percent of the net premium by which the gross premium exceeds the net premium. For example, if a company reckons, according to the table of 17 English life insurance companies with a 4.5% discount (in many cases the interest rate is even higher, thus reducing the net premium and increasing the loading) and a loading of 30% of net premium. Then the profit at the moment an insurance policy is sold is for issue age 20 6.63% of the sum insured:

Age	Profit
20	6.63% of the insured sum
30	8.16% of the insured sum
37	9.60% of the insured sum
40	10.34% of the insured sum
50	13.38% of the insured sum

60	17.02% of the insured sum
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If in a year such a company sells insurance totally 5 million Thaler (on the average age for issuing of 37) then the profit is no less than 480,000 Thaler.

The first way therefore cannot be accepted because during the time when the liabilities of the bank (reserve and mortality) are based on a net premium as large as the gross premium, nothing is left to the company for paying operating expenses. If the company has significant equity, then it can meet the costs of a small expansion in the business by using the interest on this equity. If the expansion of the business exceeds a certain limit, then the interest on the most immense equity will not be adequate. One can therefore not accept a method which does not offer increased revenues to cover the costs of an increase in business. In banks using rational assumptions the margins available to cover operating costs and beyond that for generating profits, increase as a specified proportion of premium income and then as a known proportion of the premium reserve, in that any amounts above the assumed percentage (which is never higher than 3.5% in solid companies) for interest income are not added to the reserve, but rather to the administrative fund or profits. As a rule the latter revenue is completely lacking for companies using irrational assumptions, since they usually assume such a high rate of interest that the interest on the reserve funds is less than the assumed.

The method of basing the reserve on gross premiums suffers from the same problem, since it leaves nothing over from future premium income to pay administrative costs. It also entails further deleterious effects that would lead to the rapid and sure ruin of a company. Next, we want to compare the amount of the reserve based on gross premiums with the amount of the reserve calculated rationally. Table I shows four columns with the reserve for issue ages 30, 40, and 50 with a 4.5% discount rate and with a loading of 30% of premium. It can be seen here that the reserve is negative in the first 5-7 years, later the reserve becomes positive and if the insurance stays in force till maturity the reserve finally grows to the full amount of the sum insured. Table II shows, in column 7, the reserves the bank puts aside for a number of 30-year-olds insured together for 500,000 Thaler, if the given insurance policies are followed to maturity. Tables III and IV show the corresponding results for such persons age 40 or 50 at issue. Column 8 and 9 of Tables II-IV also include the active and passive postings, which are popular in English closing accounts. The difference between the figures in columns 8 and 9 gives the reserve in column 7. Table V summarized the results of the three previous tables. If one adds up vertically the figures in the individual columns, one obtains the results which the insurance achieves, if such insurance patterns materialize every year. Taken together, the totals from the three tables results in the figures in Table V. The premium base for the premium income (Column 3) is the premium at 4.5% loaded by 30%. Columns 8 and 9 have been omitted here, but they can be derived by addition from Tables II-IV.

Table V shows that the total reserve becomes positive only in the 13th year and that after 60 years, when a stationary situation results under the assumption given, the reserve is about 4 million less than the reserve based on rational assumptions in columns 4, 5 and 6. The difference is about 10% of the sum insured, more than 30% of the rational reserve and about 350% of premium income. These proportions become still more colossal if one assumed increasing business as in Table VI. Here, after 30 years, the difference between the reserve based on gross premiums and the rationally calculated reserve is about 9 million Thalers, i.e. more than 50% of the rational reserve.

In order to study further the consequences that gross premium calculations have on the other development of a business, let us now assume that no insurance policy lapses before maturity. If the company does the same amount of business for some time, then, and this is a natural occurrence, profits are reduced from year to year. For example, if we use the figures in Table V and assume there are funds available from which the company can draw the expected future profits as long as premium income does not suffice, assume further all funds are invested at 4.5%, and reckon the agent's commission at half the first year premium and only 1% of all ensuing premiums, and other administrative costs at 20,000 Thaler a year increased by 0.1% of the sum insured, then the business realizes a profit as is shown in the following table:

[ms. P28]

Business Year	Profits	Premiums Collected	Profits in % of Premiums Collected
1	120,000	43,800 Thaler	274%
5	111,400	213,900	52%
10	101,700	413,800	25%
15			
20			
30			
60			

The profit decreases about 58% in the course of time. It's a necessary consequence that with the same amount of business the profits will decrease, since all the profit an insurance policy can yield is used up during the year the insurance is sold. If business continues at the same rate, then year after year the same profit would result if the business expenses remain the same. But the latter increase as business expands and consequently profits will decrease. Conversely, if a company wants to achieve the same profit, then it must write more insurance each subsequent year if the profit is not to sink below a specified percentage of premium income. (In the latter case, the profits must rise in proportion to premium income from a specified time).

This phenomenon which characterizes the whole system as unhealthy, even occurs where insurance stays in force till maturity. But what happens if a large proportion of insurance continues to lapse? The chief evil of the system surfaces. The majority of lapses occur with newer insurance with policies where the reserve is still negative. With the lapse of such insurance the negative reserve, or rather the overstated asset account, is dropped. This loss can only be covered by the company with new insurance policies, which of course brings negative reserves and thus arises the further need for growth, as shown above, to achieve the yearly profit. Since with increasing growth the number of lapses increases, business must increase ever more rapidly, and the faster it grows, so much the faster and faster it must grow again. Since superhuman energy is necessary to fulfill these demands, the sad consequences of the system become evident. By virtue of reckoning with gross premiums where the reserve, the liability of the bank to the insured, becomes so small as even to appear conversely as the liability of the insureds, the company quickly gets in trouble because of inadequate growth (not even taking into account the losses due to the high rate of assumed interest). Next, in order to produce the expected profit, then in order to cover a deficit no longer under control, they are forced to post all sorts of dubious accounting entries, which the public can hardly evaluate. Finally, when this maneuver no longer works, they try to get rid of the entire business by letting another company take it over. It is almost impossible to believe that company can get rid of such an ailing business, but experience shows that in England so many mergers and business transfers take

place. How a company can be drawn into such a sad deal one cannot understand by German standards. Such companies can only speculate that that in the coming years premium income will exceed the disbursements for mortality and that they are immediately able to book the excess portion of premium income under some form of profit. Finally, when this cannot continue, and the company fails, it does not matter if many insureds remain unprotected if the leaders of the business, including shareholders among them have enjoyed personal profits. They are not hindered in any way from starting a new business under a new name. Truly, the most poorly structured death fund, which takes contributions according to need, is preferable to such life insurance institutions. There, when the fund gives out, the members generally have contributed the least possible amount and in fact too little when it goes bankrupt. Here they have paid sufficient contributions and must finally leave empty handed.

If an insured who has been insured a long time, so that an individually allocated positive reserve would exist, wants to terminate his insurance, then company could justifiably allow a surrender value but in reality, the money is lacking because the negative reserve of a younger insured offsets the positive reserve and both reserves together cancel each other partially or completely. That is why one in the insurance agreements of such companies looks in vain for a clause requiring a non-forfeiture value. Some companies declare openly without giving the real reason that universal non-forfeiture would be to the disadvantage of the company.

Of course, if one needs to pay out more than one has, it can do the company harm.

There are yet more consequences of the unhealthy system of calculating [reserves] with the gross premium that can be cited. We only want to mention the following. If a company suddenly cannot insure new people by the rational reserve method, the reserve still increases for a time, if a stationary situation has not already been achieved. But in any case, the reserve grows less than if new policies are being added. With the gross premium system the opposite is true. If no new policies are added, the reserve increases more during many years than if new policies were being added.

[ms page 30-31]

Now some companies, which obviously use the gross premium method, declare that they put aside a sufficient amount for administrative costs from the gross premiums when they calculate the value of future premium payments. Granted now that the value of the sum insured is not only calculated according to the same actuarial mortality table, but also with the same interest rate as the value of the future premium payments (a higher interest rate for reckoning the worth of the sum insured, and some companies calculate this way, will reduce this value and thereby augment the setting aside of part of the gross premium for administrative costs, if more serious consequences do not occur) – so then the setting aside of the entire loading would lead to a rational basis for the reserve. But generally, premiums have colossal loadings, some 40% of the net premium and more. If 5% is put aside, then there remains for the calculation of future premium income, applicable premiums with still such high loadings that the reserve for an individual insurance policy here also takes several years to turn a negative into a positive. If here the annual profit was the same for the same amount of business without any lapses, as measured against the sum insured, not as a percentage of premium, so must the yearly growth grow in order to maintain the same profit if some of the insurance policies lapse, and since with increasing growth the number of lapsing insurance policies grows, so must the increase grow that much more. If the annual profit should not go below a certain percentage of premium income, so must the year's increase grow still faster. The existence of the company is bound as above to the demand that the yearly increase grows faster every year. The same consequences result in this

case as when the reserve is based on the gross premium, only they take longer and are less serious.

If we summarize the foregoing, then the error of basing the reserve on gross premiums that are higher than the net premium for risk, but generally equal to the premium actually charged the insured, consists in the following:

- 1) With the closing of an insurance policy an artificial profit is booked which in actuality need not be realized and in so many cases is not realized.
- 2) The premium reserve for an individual insurance policy is simply too low; moreover, the negative reserves of new policies offsets the positive reserves partially or completely.
- 3) The profit mentioned in point 1 is the only, or at least the main source:
 - a). for covering administrative costs
 - b). for covering future premature lapses of insurance policies with negative reserves
 - c). for the yearly profit

Since the first two items are necessary expenditures and even increasing ones, but since the profit mentioned under (1) cannot always keep growing, or what amounts to the same thing, since the yearly growth cannot keep increasing enough each year, then the yearly profit mentioned under (c) must sooner or later decrease and finally must produce a deficit and the bank becomes insolvent.

Notes:

1. There are other reasons for recommending a closing commission. For example, how will an agent who does considerable business in a short time be compensated using the usual service fee if he is forced to give up his business as a result of death or other circumstances.
2. If one tries to establish the amortization of the advanced commission account on a rational basis, particularly so that it is free from the second and third instances of the aforementioned difficulties mentioned above, then one naturally arrives at the proposed method.
3. It is assumed that the amount booked as an asset is the same for each issue age with the proportion of the premium varying by issue age, but with the proportion of sum insured staying constant. If the amount by which the first year net premium is less than the renewal net premium is a fixed percentage of the net premium, the formula becomes somewhat different and the following comments do not apply word for word.
4. Since the maximum amounts increase in a similar relationship to the premium, it seems advantageous to calculate the closing costs according to the percentage of the first year's premium.

Issue Age	Maximum Rate of Closing Costs as
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	Percentage of First Year Premium
20	50.1
30	65.0
40	61.1
50	60.2
60	58.2

Here one sees that the maximum percentage is not excessive.

5. If an insurance policy is not a full year old at the end of the business year, then the reserves at the end of the business year is calculated according to the formula for fractional reserves.
6. Not counting the other consequences which such assumptions must have on the financial development, the following situation results, in order not to use a stronger expression undeserved because this point can only be realized if the policyholder keeps that policy in effect properly until the end of term, or can be forced to keep it until then. But though the bank knows that a number of policies will lapse it still books a profit.
7. Here the profit is greater than the premium income. Often the companies who calculate this way have more profit than they process in hard cash or assets. Therefore, one finds frequently huge book profits held back for distribution later.
8. The profit varies directly with the loading. Therefore in England nearly all companies are either mutual or mixed. Pure stock companies would have to load the net premium as little as possible.

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August Zillmer

Table I

Reserves for an insurance policy of 100 Thaler, payable upon the death of the insured or the attainment of age 90 with yearly premium payments.

TIME POLICY IS IN EFFECT IN YEARS	ISSUING AGE 30 RESERVES ON A 3.5% NET PREMIUM THE FIRST YEAR PREMIUM IS REDUCED BY			4.5% NET PREMIUM BASIS WITH A 30% LOADING	ISSUING AGE 40 RESERVES ON A 3.5% NET PREMIUM THE FIRST YEAR PREMIUM IS REDUCED BY			4.5% NET PREMIUM BASIS WITH A 30% LOADING	ISSUING AGE 50 RESERVES ON A 3.5% NET PREMIUM THE FIRST YEAR PREMIUM IS REDUCED BY			4.5% NET PREMIUM BASIS WITH A 30% LOADING	TIME POLICY IS IN EFFECT IN YEARS					
	0% OF THE SUM INSURED	1.00% THE SUM INSURED	1.25% THE SUM INSURED		0% OF THE SUM INSURED	1.00% THE SUM INSURED	1.25% THE SUM INSURED		0% OF THE SUM INSURED	1.00% THE SUM INSURED	1.25% THE SUM INSURED							
	1	1.020	0.036		-0.211	-7.230	1.553		0.569	0.522	-8.813			2.281	1.304	1.060	-11.093	1
	2	2.083	1.104		0.859	-6.288	3.150		2.182	1.439	-7.275			4.586	3.632	3.393	-8.708	2
3	3.168	2.200	1.958	-5.306	4.793	3.841	3.603	-5.678	6.915	5.984	5.751	-6.296	3					
4	4.285	3.328	3.089	-4.289	6.481	5.546	5.312	-4.033	9.262	8.355	8.128	-3.852	4					
5	5.435	4.489	4.253	-3.236	8.205	7.287	7.058	-2.337	11.629	10.745	10.824	-1.373	5					
6	6.619	5.685	5.452	-2.142	9.963	9.063	8.898	-0.595	14.012	13.152	12.937	1.136	6					
7	7.836	6.914	6.684	-1.013	11.751	10.869	10.618	1.181	16.497	15.571	15.362	3.670	7					
8	9.090	8.181	7.954	0.160	13.563	12.699	12.483	2.999	18.816	18.004	17.801	6.234	8					
9	10.881	9.485	9.261	1.377	15.404	14.558	14.347	4.850	21.231	20.446	20.219	8.818	9					
10	11.708	10.825	10.604	2.638	17.267	16.440	16.220	6.740	23.659	22.896	22.704	11.425	10					
11	13.080	12.211	11.934	3.948	19.154	18.846	18.143	8.661	26.084	25.345	25.160	14.040	11					
12	14.489	13.634	13.420	5.306	21.061	20.272	20.074	10.614	28.603	27.788	27.609	16.669	12					
13	15.940	15.099	14.889	6.715	22.938	22.218	22.025	12.598	30.911	30.220	30.047	19.294	13					
14	17.430	16.604	16.398	8.168	24.929	24.178	23.991	14.607	33.307	32.640	32.473	21.918	14					
15	18.952	18.142	17.839	9.665	26.888	26.157	25.974	16.645	35.694	35.041	34.874	24.537	15					
20	26.953	26.229	26.040	17.676	36.840	36.208	36.051	27.108	47.179	46.651	46.506	37.342	20					
25	35.448	34.802	34.641	26.420	46.790	46.258	46.125	37.951	57.833	57.411	57.317	49.419	25					
30	44.235	43.677	43.538	35.710	56.299	55.862	55.753	48.479	67.720	67.397	67.371	60.892	30					
35	53.020	52.550	52.433	45.228	65.114	64.765	64.678	58.431	78.253	100,000	100,000	73.332	35					
40	61.416	61.030	60.934	54.321	73.294	73.027	72.98	67.843	100,000			100,000	40					
45	69.199	68.891	68.814	63.309	82.008	81.828	81.783	78.072					45					
50	76.421	76.185	76.126	71.615	100,000	100,000	100,000	100,000					50					

Table II (A)												
Over view of business result if a company insures a group of 30 year olds, each for the same amount, for a 500,000 total.												
The yearly premium is 2 Thr. 2 Sgr. 9 PFg.												
	1	2	3	4	5	6	7	8	9			
	AMOUNT OF SUMS INS AT END OF YEAR	AMOUNT DUE IN CLAIMS	PREMIUMS COLLECTED	RESERVES ON A				ASSET ELEMENT VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7	LIABILITY ELEMENTS VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7			
				3 1/2% NET PREMIUM BASIS IF THE FIRST YEAR PREMIUM IS REDUCED BY			4 1/2% NET PREMIUM BASIS WITH A 30% LOADING					
YEARS				0 % OF THE SUM INSURED	1% OF THE SUM INSURED	1 1/4% OF THE SUM INSURED				YEARS		
1	495,800	4,200	10,458	5,100	193	-1,034	-35,900	173,800	137,900	1		
2	491,500	4,300	10,370	10,200	5,387	4,184	-30,900	170,800	139,900	2		
3	487,200	4,300	10,281	15,400	10,682	9,603	-25,800	167,700	141,900	3		
4	482,900	4,300	10,192	20,700	16,078	14,923	-20,700	164,600	143,900	4		
5	478,500	4,400	10,100	26,000	21,475	20,344	-15,500	161,500	146,000	5		
6	474,100	4,400	10,009	31,400	26,973	25,867	-10,200	158,300	148,100	6		
7	469,600	4,500	9,916	36,800	32,472	31,390	-4,800	155,100	150,300	7		
8	465,000	4,600	9,822	43,200	38,073	37,016	700	151,800	152,500	8		
9	460,400	4,600	9,726	47,800	43,674	42,643	6,400	148,400	154,800	9		
10	455,700	4,700	9,630	53,300	49,276	48,270	12,000	145,100	157,100	10		
11	451,000	4,700	9,533	59,000	55,080	54,100	17,800	141,600	159,400	11		
12	446,200	4,800	9,434	64,600	60,784	59,830	23,700	138,100	101,800	12		
13	441,400	4,800	9,334	70,400	66,690	65,762	29,600	134,600	164,200	13		
14	436,400	5,000	9,232	76,100	72,497	71,596	35,700	131,000	166,700	14		
15	431,300	5,100	9,128	81,700	78,204	77,330	41,700	127,400	169,100	15		
16	426,000	5,300	9,021	87,300	83,913	83,066	47,700	123,700	171,400	16		
17	420,600	5,400	8,911	93,000	89,724	88,905	53,700	119,900	173,600	17		

Table II B)										
Over view of business result if a company insures a group of 30 year olds, each for the same amount, for a 500,000 total.										
The yearly premium is 2 Thlr. 2 Sgr. 9 PFg.										
	1	2	3	4	5	6	7	8	9	
	AMOUNT OF SUMS INS AT END OF YEAR	AMOUNT DUE IN CLAIMS	PREMIUMS COLLECTED	RESERVES ON A				ASSET ELEMENT VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7	LIABILITY ELEMENTS VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7	
				3 1/2% NET PREMIUM BASIS IF THE FIRST YEAR PREMIUM IS REDUCED BY			4 1/2% NET PREMIUM BASIS WITH A 30% LOADING			
YEARS				0 % OF THE SUM INSURED	1% OF THE SUM INSURED	1 1/4% OF THE SUM INSURED				YEARS
24	375,400	7,300	8,005	126,600	124,112	123,490	92,400	92,500	184,900	24
25	367,800	7,600	7,852	130,400	128,026	127,432	97,100	88,500	185,600	25
26	359,800	8,000	7,692	133,800	131,540	130,975	101,600	84,400	186,000	26
27	351,500	8,300	7,526	136,900	134,754	134,217	105,800	80,300	186,100	27
28	342,800	8,700	7,352	139,500	137,467	136,959	109,500	76,300	185,800	28
29	333,700	9,100	7,170	141,700	139,780	139,300	112,900	72,200	185,100	29
30	324,300	9,400	6,981	143,400	141,591	141,139	115,800	68,200	184,000	30
31	314,500	9,800	6,784	144,700	143,002	142,578	118,300	64,100	182,450	31
32	304,200	10,300	6,578	145,300	143,711	143,313	120,200	60,200	180,499	32
33	293,500	10,700	6,363	145,400	143,919	143,549	121,500	56,200	177,700	33
34	282,400	11,100	6,140	144,800	143,424	143,080	122,400	52,300	174,700	34
35	270,900	11,500	6,908	143,600	142,327	142,009	122,500	48,500	171,000	35
36	259,000	11,900	5,666	141,800	140,628	140,335	122,000	44,800	166,800	36
37	246,600	12,400	5,417	139,200	138,126	137,857	120,900	44,100	162,000	37
38	233,900	12,700	5,159	135,900	134,920	134,675	119,000	37,600	156,600	38
39	220,900	13,000	4,893	132,000	131,111	130,889	116,400	34,200	150,600	39
40	207,600	13,300	4,621	127,500	126,699	126,499	113,200	30,900	144,100	40

Table II [C]												
Over view of business result if a company insures a group of 30 year olds, each for the same amount, for a 500,000 total.												
The yearly premium is 2 Thlr. 2 Sgr. 9 PFg.												
	1	2	3	4	5	6	7	8	9			
	AMOUNT OF SUMS INS AT END OF YEAR	AMOUNT DUE IN CLAIMS	PREMIUMS COLLECTED	RESERVES ON A				ASSET ELEMENT VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7	LIABILITY ELEMENTS VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7			
				3 1/2% NET PREMIUM BASIS IF THE FIRST YEAR PREMIUM IS REDUCED BY			4 1/2% NET PREMIUM BASIS WITH A 30% LOADING					
YEARS				0 % OF THE SUM INSURED	1% OF THE SUM INSURED	1 1/4% OF THE SUM INSURED				YEARS		
47	113,300	13,000	2,642	81,700	81,384	81,305	75,600	12,300	87,900	47		
48	100,600	12,700	2,369	74,000	73,734	73,667	68,800	10,400	79,200	48		
49	88,500	12,100	2,105	66,400	66,179	66,124	61,900	8,700	70,600	49		
50	77,100	11,400	1,852	58,900	58,178	58,672	55,200	7,100	62,300	50		
51	66,200	10,900	1,611	51,500	51,353	51,326	48,500	5,800	54,300	51		
52	56,200	10,000	1,385	44,600	44,484	44,455	42,100	4,600	46,700	52		
53	47,000	9,200	1,202	38,000	37,910	37,887	36,000	3,600	39,600	53		
54	38,700	8,300	983	31,900	31,832	31,815	30,500	2,700	33,200	54		
55	31,400	7,300	810	26,400	26,350	26,337	25,300	2,000	27,300	55		
56	25,000	6,400	657	21,500	21,465	21,456	20,700	1,400	22,100	56		
57	19,400	5,600	522	17,100	17,077	17,071	16,600	900	17,500	57		
58	14,700	4,700	406	13,400	13,387	13,384	13,100	500	13,600	58		
59	10,800	3,900	307	10,200	10,194	10,193	10,100	200	10,300	59		
60	0	10,800	216	0	0	0	0	0	0	60		

Table III (A)										
Over view of business result if a company insures a group of 40 year olds, each for the same amount, for a 750,000 total.										
The yearly premium is 2 Thlr 29Sgr. 3Pfg.										
	1	2	3	4	5	6	7	8	9	
	AMOUNT OF SUMS INS AT END OF YEAR	AMOUNT DUE IN CLAIMS	PREMIUMS COLLECTED	RESERVES ON A				ASSET ELEMENT VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7	LIABILITY ELEMENTS VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7	
				3 1/2% NET PREMIUM BASIS IF THE FIRST YEAR PREMIUM IS REDUCED BY			4 1/2% NET PREMIUM BASIS WITH A 30% LOADING			
YEARS				0 % OF THE SUM INSURED	1% OF THE SUM INSURED	1 1/4% OF THE SUM INSURED				YEARS
1	742,200	7,800	22,062	11,500	4,193	2,366	-71,400	327,800	256,400	1
2	734,300	7,900	21,833	23,100	15,988	14,210	-59,500	319,700	260,200	2
3	726,300	8,000	21,602	34,800	27,885	26,156	-47,400	311,500	264,100	3
4	718,200	8,100	21,366	46,600	39,884	38,205	-35,200	303,200	268,000	4
5	709,800	8,400	21,126	58,300	51,785	50,156	-22,900	294,800	271,900	5
6	701,100	8,700	20,879	69,900	63,588	62,010	-10,500	286,200	275,700	6
7	692,100	9,000	20,624	81,300	75,192	73,665	1,600	277,600	279,200	7
8	682,700	9,400	20,359	92,600	86,699	85,234	13,900	268,800	282,700	8
9	673,000	9,700	20,084	103,700	98,007	96,583	26,000	259,900	285,900	9
10	662,900	10,100	19,798	114,500	109,016	107,645	37,900	250,900	288,800	10
11	652,300	10,600	19,499	124,900	119,626	118,307	49,700	241,800	291,500	11
12	641,300	11,000	19,188	135,100	130,038	128,773	61,300	232,600	293,900	12
13	629,800	11,500	18,864	144,800	139,950	138,737	72,500	223,400	295,900	13
14	617,700	12,100	18,526	154,000	149,363	148,204	83,300	214,100	297,400	14
15	605,500	12,500	18,172	162,700	158,275	157,169	93,800	204,700	298,500	15
16	592,100	13,100	17,803	170,900	166,688	165,627	103,900	195,300	299,200	16
17	578,400	13,700	17,417	128,400	174,400	173,400	113,300	185,900	299,200	17

18	564,100	14,300	17,015	185,200	181,411	180,464	122,300	176,500	298,800	18
19	549,200	14,900	16,595	191,300	187,721	186,826	130,500	167,100	297,600	19
20	533,700	15,500	16,157	196,600	193,229	192,387	138,100	157,800	295,900	20
21	517,500	16,200	15,700	201,000	197,835	197,044	144,900	148,500	293,400	21
22	500,700	16,800	15,224	204,600	201,639	200,899	150,900	139,200	290,100	22
23	483,100	17,600	14,728	207,000	204,239	203,549	155,800	130,100	285,900	23
24	464,800	18,300	14,210	208,300	205,735	205,094	155,900	121,100	281,000	24

Table IV												
Over view of business result if a company insures a group of 50 year olds, each for the same amount, for a 250,000 total.												
The yearly premium is 4 Thlr. 15 Sgr. 3 PFg.												
	1	2	3	4	5	6	7	8	9			
	AMOUNT OF SUMS INS AT END OF YEAR	AMOUNT DUE IN CLAIMS	PREMIUMS COLLECTED	RESERVES ON A				ASSET ELEMENT VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7	LIABILITY ELEMENTS VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7			
				3 1/2% NET PREMIUM BASIS IF THE FIRST YEAR PREMIUM IS REDUCED BY			4 1/2% NET PREMIUM BASIS WITH A 30% LOADING					
YEARS				0 % OF THE SUM INSURED	1% OF THE SUM INSURED	1 1/4% OF THE SUM INSURED				YEARS		
1	246,000	4,000	11,271	5,600	3,196	2,595	-27,300	139,800	112,500	1		
2	241,900	4,100	11,091	11,100	8,792	8,215	-21,100	134,500	113,400	2		
3	237,500	4,400	10,904	16,400	14,189	13,636	-14,900	129,100	114,200	3		
4	233,000	4,500	10,709	21,600	19,486	18,957	-9,000	123,800	114,800	4		
5	228,200	4,800	10,504	26,500	24,483	23,979	-3,100	118,300	115,200	5		
6	223,300	4,900	10,290	31,300	29,380	28,900	2,600	112,900	115,500	6		
7	218,100	5,200	10,067	35,700	33,876	33,420	8,000	107,500	115,500	7		
8	212,700	5,400	9,835	40,000	38,273	37,841	13,300	102,000	115,300	8		
9	207,100	5,600	9,592	43,900	42,268	41,860	18,300	96,600	114,900	9		
10	201,300	5,800	9,339	47,600	46,063	45,679	23,000	91,200	114,200	10		
11	195,200	6,100	9,075	50,900	49,457	49,096	27,400	85,800	113,200	11		
12	188,800	6,400	8,800	53,800	52,450	52,112	31,500	80,500	112,000	12		
13	182,200	6,600	8,513	56,300	55,041	54,726	35,200	75,200	110,400	13		
14	175,300	6,900	8,214	58,400	57,231	56,939	38,400	70,000	108,400	14		
15	168,100	7,200	7,903	60,000	58,919	58,649	41,300	64,900	106,200	15		
16	160,700	7,400	7,580	61,100	60,104	59,855	43,600	59,900	103,500	16		
17	153,100	7,600	7,246	61,800	60,887	60,659	45,500	55,000	100,500	17		

Table V									
Over view of business results which a company tries to achieve, if it insures people for									
1.5 million per year. (30 year olds, 40 year olds for .75 million and 50 year olds for .25 million.)									
	1		3	4	5	6	7	8	
	AMOUNT OF SUMS INS AT END OF YEAR	AMOUNT DUE IN CLAIMS	PREMIUMS COLLECTED	RESERVES ON A				ASSET ELEMENT VALUE OF ALL FUTURE PREMIUMS IN COLUMN 7	
				3 1/2% NET PREMIUM BASIS IF THE FIRST YEAR PREMIUM IS REDUCED BY			4 1/2% NET PREMIUM BASIS WITH A 30% LOADING		
YEARS				0 % OF THE SUM INSURED	1% OF THE SUM INSURED	1 1/4% OF THE SUM INSURED			
1	1,484,000	16,000	43,800	22,200	7,600	3,900	-134,600	1	
2	2,951,700	32,300	87,100	66,600	37,700	30,500	-246,100	2	
3	4,102,700	49,000	129,900	133,200	90,500	79,800	-334,200	3	
4	5,836,800	65,900	172,100	222,100	166,000	151,900	-339,100	4	
5	7,253,300	83,500	213,900	332,900	263,700	246,400	-440,600	5	
6	8,651,800	101,500	255,000	465,500	383,600	363,200	-458,700	6	
7	10,031,600	120,200	295,700	619,300	525,200	501,600	-453,900	7	
8	11,392,000	139,600	335,700	794,200	688,200	661,700	-426,000	8	
9	12,732,500	159,500	375,100	989,600	872,200	842,800	-375,300	9	
10	14,052,400	180,100	413,800	1,205,000	1,076,500	1,044,400	-302,400	10	
11	15,350,900	201,500	451,900	1,439,800	1,300,700	1,265,900	-207,500	11	
12	16,627,200	223,700	489,400	1,693,300	1,544,000	1,506,600	-91,000	12	
13	17,880,600	246,600	526,100	1,964,800	1,805,600	1,765,900	46,300	13	
14	19,110,000	270,600	562,100	2,353,300	2,084,700	2,042,600	203,700	14	
15	20,314,600	295,400	597,300	2,557,700	2,380,100	2,335,700	380,500	15	
16	21,493,100	321,200	631,700	2,877,000	2,690,800	2,644,300	575,700	16	

